FREE VIBRATIONS OF FRAMED STRUCTURES WITH INCLINED MEMBERS

A Thesis submitted in partial fulfillment of the requirements for the degree of

Bachelor of Technology in "Civil Engineering"

By

JYOTI PRAKASH SAMAL (108CE007)

Under the guidance of

Prof. M.R. BARIK

Department of Civil Engineering

National Institute of Technology

Rourkela-769008 (ODISHA)

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DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

ODISHA, INDIA-769008

CERTIFICATE

This is to certify that the thesis entitled "**Free Vibrations of Framed Structures with Inclined Members"**, submitted by **Jyoti Prakash Samal (108CE007)** in partial fulfillment of the requirements for the award of **Bachelor of Technology** in **Civil Engineering** during session 2011-2012 at National Institute of Technology, Rourkela is a bonafide record of research work carried out by him under my supervision and guidance.

The candidate has fulfilled all the prescribed requirements.

The thesis which is based on candidates' own work has not been submitted elsewhere for a degree/diploma.

In my opinion, the thesis is of standard required for the award of a Bachelor of Technology degree in Civil Engineering.

Place: Rourkela

Dept. of Civil Engineering
National institute of Technology

Prof. M.R. Barik
Associate Professor

ACKNOWLEDGEMENTS

On the submission of my thesis entitled "**Free Vibrations of Framed Structures with Inclined Members**", I would like to extend my gratitude & my sincere thanks to my supervisor **Prof. M.R. Barik**, Associate Professor, Department of Civil Engineering for his constant motivation and support during the course of my work in the last one year. I truly appreciate and value his esteemed guidance and encouragement from the beginning to the end of this thesis. His knowledge and guidance at the time of crisis would be remembered lifelong.

I am very thankful to **Prof. Ramakar Jha for** his valuable suggestions and comments during this project period.

I am very thankful to my teachers for providing solid background for my studies and research thereafter. They have great sources of inspiration to us and I thank them from the bottom of my hearts.

At last but not least, I would like to thank the staff of Civil engineering department for constant support and providing place to work during project period. I would also like to extend my gratitude to my friends who are with me during thick and thin.

JYOTI PRAKASH SAMAL

B.Tech (Civil Engineering)

ABSTRACT

The structures always vibrate under dynamic loadings under certain frequencies. These frequencies may resonate with natural frequencies. Hence it is important to determine the natural frequencies. Finite Element Method is a versatile numerical tool which is used for the vibration analysis of frames which are subjected to dynamic load, which is comprised of time varying loads. The Project titled "**Free Vibrations of Framed Structures with Inclined Members**", aims in finding free un-damped natural frequencies of vibrations of frames with inclined members. FEM is used to solve the problems by developing codes in MATLAB in which the Stiffness and Mass matrices of structure are constructed in MATLAB. The natural frequencies are then computed.

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Chapter-1

INTRODUCTION

1.1 GENRAL INTRODUCTION

One of the most important things engineers can do is to model the physical phenomena. Every phenomenon in nature can be described with the aids of laws of physics, or other fields in algebraic, differential or integral equation relating different quantities. “Analytical description of physical phenomena and process are called *mathematical models*.” These models are made using different assumptions of process, axioms. They are characterized by very complex differential and integral calculus.

The buildings and structures in civil engineering are subjected to static and dynamic loadings. Vibrational analysis of building is important as they are continuously subjected to dynamic loads like wind, earthquake etc. Various classical methods are present which could be used to solve these problems and now-a-days many software have also come up which help us to predict the behavior of a structure more accurately.

When a structure is given an excitation (force is only required to initiate, it doesn’t have any further role. Hence an un-forced condition), it vibrates freely and finally comes to rest due to damping. These Vibrations are called a FREE VIBRATIONS, and frequencies are called NATURAL FREQUENCIES of vibrations.

Any building or structure can be modeled into frames. These frames can be analyzed, and the behavior of the structure can be predicted. Finite Element Method is a versatile tool which can be used to mathematically model and analyze the structure. The in-plane vibrations of frames with inclined members are studied by writing codes in MATLAB and using Finite Element Method.

Chapter-2

LITRATURE REVIEW

Gladwell G.M.L (1964) presented a method to find natural frequencies and modes of un-damped free vibrations of a plane frame with rectangular grids of uniform beams in his paper. He presented a general method for finding out set of modes which could be used in Rayleigh-Ritz analysis of systems. He showed how inertia and stability matrices of modes could be found. He set up whole analysis in matrix, which was used in digital computer. He applied the method to a simple frame and compared it with exact solution.

Chang C.H (1975), in his paper, presented vibrations of inclined bars with different constraints. The frequencies of the vibrations of small amplitudes of inclined bars with different end conditions were presented along with normal functions. The application of the method of normal function expansion to the forced as well as transient vibrations of the inclined bars was also outlined.

Howson W.P, Williams F.W (1972), in their paper, presented dynamic stiffness matrix to calculate natural frequencies. They presented first four in-plane free vibrations of an H-shaped frame.

Ahmed El-Sheikh (1998), in his paper, presented two approximate analysis methods suitable for the dynamic analysis of space trusses. The methods, which were based on beam and plate analogy techniques, involved simple hand calculations. It was intended to provide easy and accurate predictions of the dynamic behavior of space trusses.

Weaver W, Eisenberger M (1981), in their paper, used FEM to get natural frequencies of vibrations of plane and space frames. They reduced the matrix by imposing axial constraints.

They programmed in digital computers for plane and space frames. The redundant constraints were identified automatically.

Chapter-3

FORMULATION OF PROBLEM

3.1 FINITE ELEMENT METHOD.

Finite Element Method (FEM) is a general and powerful numerical method. It can be applied to real world problem that involve complex physical, geometric or boundary conditions. In FEM, a given domain is studied as a collection of sub-domains. The idea is that, it is easier to represent a complex polynomial as a collection of simpler polynomials.

A geometrically complex domain Ω, is represented as collection of simple domain Ω_e, which is viewed as an independent domain in itself. 'Domain' is the geometric region over which equation is solved. Second, over each element, algebraic equation and other quantities are developed using governing equations. Third, these relationships in all elements are assembled. The steps are:-

1. Finite element discretization
2. Element equation
3. Assembly of element equation and solution,
4. Convergence and error estimate.

3.1.1 Finite Element Discretization

First, the domain is represented as a collection of 'n' subdomains. This is called *Finite Element discretization*. Each subdomain represents an element. This collection of elements is called *Finite Element Mesh.* The Elements are connected to each other at points called *Nodes.* When elements are of equal measure, it is called as *uniform mesh*; else it is called *non-uniform mesh*.

3.1.2 Element Equation

A typical element is isolated and its properties are calculated. In this project we consider **Stiffness (K)** and **Mass (M)** as elemental properties.

B = [N(ξ)^T*N(ξ)]; where $\xi = \frac{x}{l}$

$$\mathbf{K} = \left(\frac{EI}{2a3}\right)\left(\int_{-1}^{1} \boldsymbol{B}\ \boldsymbol{\partial\xi}\right)$$

$$\mathbf{M} = \boldsymbol{\rho A a}\left(\int_{-1}^{1} \boldsymbol{B}\ \boldsymbol{\partial\xi}\right)$$

3.1.3 Assembly of Elemental Equations and Solution

The Equations are put together in a meaningful way, such that they represent the global domain 'Ω'. In this project elemental Stiffness and Mass matrices are multiplied with transformation factor, so that they fit into global Stiffness and Mass matrices.

For *Solution*,

| K-ω^2*M |=0; Eigen values of diagonal elements give square of Natural frequencies

3.2 Analysis

The analysis comprises of two parts static and dynamic.

3.2.1 Static Analysis

For a single bar element

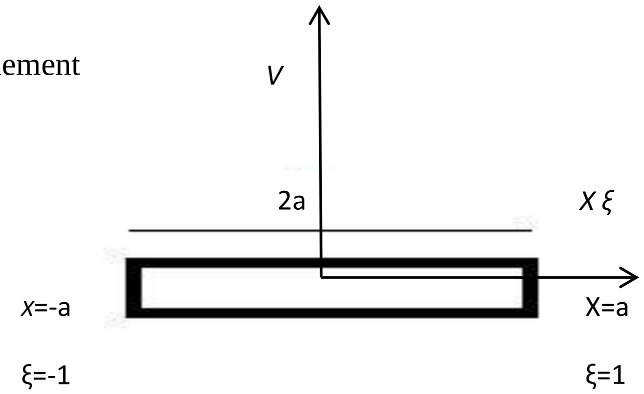


Fig 3.1

Area=A; Modulus of Elasticity= E; Length = L

u= $\alpha_1 + \alpha_2\xi$; where $\frac{x}{l} = \xi$

$u_1 = \alpha_1 - \alpha_2$, $u_2 = \alpha_1 + \alpha_2$

$\alpha_1 = \frac{1}{2}(u_1 + u_2)$

$\alpha_2 = \frac{1}{2}(u_1 - u_2)$

Solving

u= $\frac{1}{2}(1-\xi)u_1 + \frac{1}{2}(1+\xi)u_2$

$\frac{d^2u}{dx^2} = 0; \frac{d^2u}{d\xi^2} = 0;$

$N_1(\xi) = \frac{1}{2}(1-\xi)$ $N_2(\xi) = \frac{1}{2}(1+\xi)$

$$u= [N_1(\xi)\ N_2(\xi)]\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

N(ξ) – shape function

$$U_e= \frac{1}{2}\int_{-a}^{a} EA\ (\frac{\partial u}{\partial x})^2 dx$$

$$U_e= \frac{1}{2}\{U\}_e{}^T \frac{EA}{a}\int_{-1}^{1} N'(\xi)^T N'(\xi)\ d\xi\ \{U\}_e$$

$$K_e=\frac{EA}{a}\int_{-1}^{1} N'(\xi)^T N'(\xi)\ d\xi$$

$$T_e= \frac{1}{2}\int_{-a}^{a} \rho A\ (\frac{\partial u}{\partial t})^2 dx$$

$$T_e= \frac{1}{2}\{U\}_e{}^T \rho Aa\int_{-1}^{1} N'(\xi)^T N'(\xi)\ d\xi\ \{U\}_e$$

$$M_e= \rho Aa\int_{-1}^{1} N'(\xi)^T N'(\xi)\ d\xi$$

$$M_e=\rho Aa \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$K_e= \frac{(AE)}{a} * \begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$$

For a single beam element

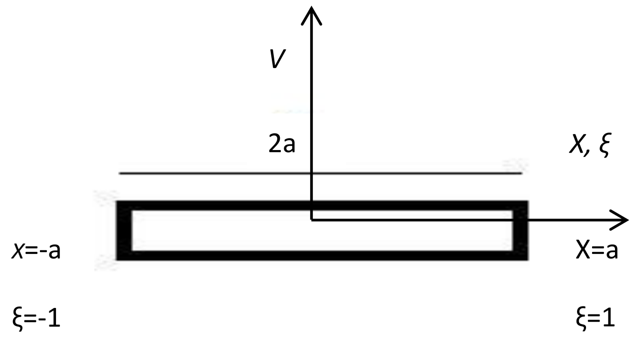


Fig 3.2

$$v=\alpha_1 + \alpha_2\xi + \alpha_3\xi^2 + \alpha_4\xi^4$$

$$v= [1 \quad \xi \quad \xi^2 \quad \xi^3]\,[\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4]^T$$

$N(\xi)=[\ \xi^3/4 - (3\xi)/4 + 1/2,\ (a\xi^3)/4 - (a\xi^2)/4 - (a\xi)/4 + a/4,\ - \xi^3/4 + (3\xi)/4 + 1/2,\ (a\xi^3)/4 + (a\xi^2)/4 - (a\xi)/4 - a/4]$

$$U_e= \frac{1}{2}\int_{-a}^{a} EI_z \left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx$$

$$T_e= \frac{1}{2}\int_{-a}^{a} \rho A \left(\frac{\partial v}{\partial t}\right)^2 dx$$

$$K_e= \frac{EI_z}{a^3}\int_{-1}^{1} [N''(\xi)]^T [N''(\xi)]\ d\xi$$

$$M_e= \rho A a\int_{-1}^{1} [N'(\xi)]^T [N'(\xi)]\ d\xi$$

Area=A; Modulus of Elasticity= E; Length = L; I= Second Moment of Area

$$K = (EI) * \begin{pmatrix} 12/L^3, & 6/L^2, & -12/L^3, & 6/L^2 \\ 6/L^2, & 4/L, & -6/L^2, & 2/L \\ -12/L^3, & -6/L2, & 12/L^3, & -6/L^2 \\ 6/L^2, & 2/L, & -6/L^2, & 4/L \end{pmatrix}$$

Taking axial displacements in to account

$$K= \begin{pmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0; \\ 0 & 12*EI/L^3 & 6*EI/L^2 & 0 & -12*EI/L^3 & 6*EI/L^2; \\ 0 & 6*EI/L^2 & 4*EI/L & 0 & -6*EI/L^2 & 2*EI/L; \\ -EA/L & 0 & 0 & EA/L & 0 & 0; \\ 0 & -12*EI/L^3 & -6*EI/L^2 & 0 & 12*EI/L^3 & -6*EI/L^2; \\ 0 & 6*EI/L^2 & 2*EI/L & 0 & -6*EI/L^2 & 4*EI/L \end{pmatrix}$$

For a Beam Element Mass matrix;

$$M=\rho A*\begin{pmatrix} L/3 & 0 & 0 & L/6 & 0 & 0 \\ 0 & (13*L)/35 & (11*L^2)/210 & 0 & (9*L)/70 & -(13*L^2)/420 \\ 0 & (11*L^2)/210 & L^3/105 & 0 & (13*L^2)/420 & -L^3/140 \\ L/6 & 0 & 0 & L/3 & 0 & 0 \\ 0 & (9*L)/70 & (13*L^2)/420 & 0 & (13*L)/35 & -(11*L^2)/210 \\ 0 & -(13*L^2)/420 & -L^3/140 & 0 & -(11*L^2)/210 & L^3/105 \end{pmatrix}$$

ρ = Density of element

A= Area of cross Section

L= Length of element

3.2.2 Dynamic Analysis

According to Newton’s law f=m*a.

⇨ k*x=m*a

⇨ k*x-m*a=0; since x= a*sin(ω.t)

⇨ k*x- ω^2 *m*x=0

⇨ (k-ω^2m)x=0; x not equal to zero hence

In Matrix form

⇨ |([K]-ω^2*[M]|=0

Square root Eigen Values of diagonal elements of Stiffness and Mass Matrices gives *Natural Frequencies of Free Vibrations.*

When Damping is there;

$$[K]*\{X\}-[C]*\{V\}-[M]*\{A\} =0; \text{ where } \{V\} =\{X\}^{\cdot}$$

⇨ $|([K]-\omega*[C]-\omega^2*[M]|=0$

C= Damping Coefficient matrix

3.3 Assembling of Equations and Solutions

3.3.1 Transformation

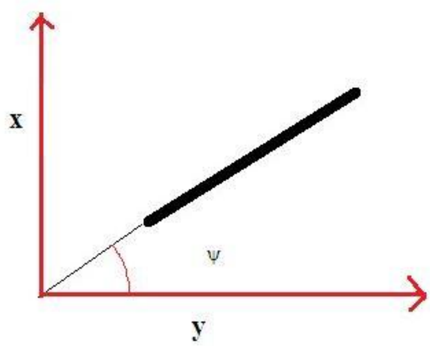


Fig 3.3

The element is inclined at an angle ‘ψ’ with horizontal in global coordinate system.

Cos ψ = C; Sin ψ = S

Transformation Matrix:-

$$L=\begin{pmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\mathbf{K_g= L'*K*L}$

$\mathbf{M_g=L'*M*L}$**; Where** $\mathbf{M_g}$ and $\mathbf{K_g}$ are Mass and Stiffness Matrixes of element in global coordinates

3.3.2 Assembling

Global Degrees of freedom = 3* number nodes

Element Nodes = e_i, e_j;

A= $(e_i-1)*3+1$;

B= $(e_j-1)*3+1$;

[A A+1 A+2 B B+1 B+2] = Element Dof

$\mathbf{K_G}$ **(Element Dof, Element Dof) =** $\mathbf{K_g}$

$\mathbf{M_G}$ **(Element Dof, Element Dof) =** $\mathbf{M_g}$

Solution:-

- ⇨ $|([K]-\omega^2*[M]|=0$

Square roots of Eigen values give *Natural Frequencies*.

Chapter-4

COMPUTER PROGRAM

Fig 4.1 Flow Chart

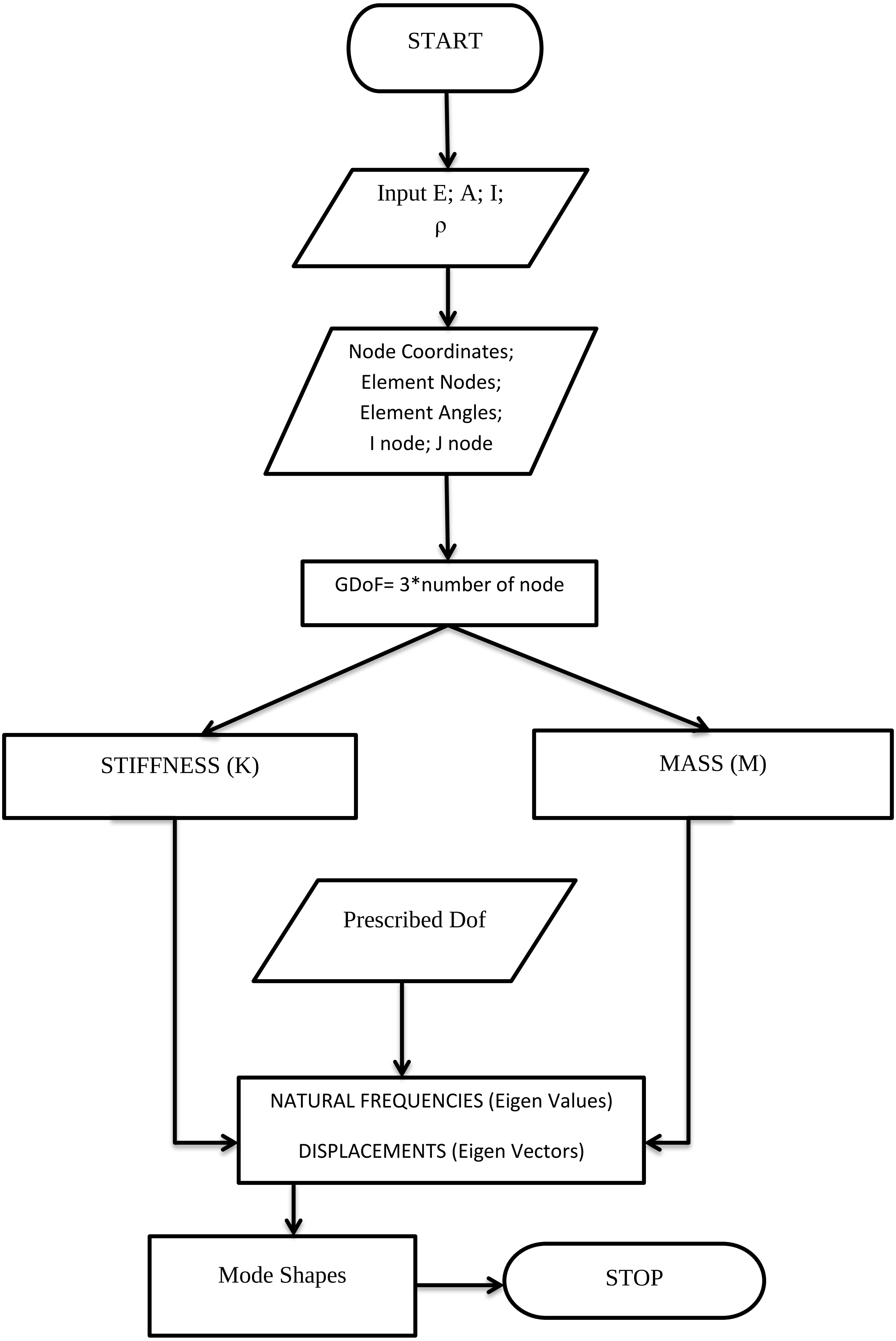


Fig 4.2 Function Stiffness

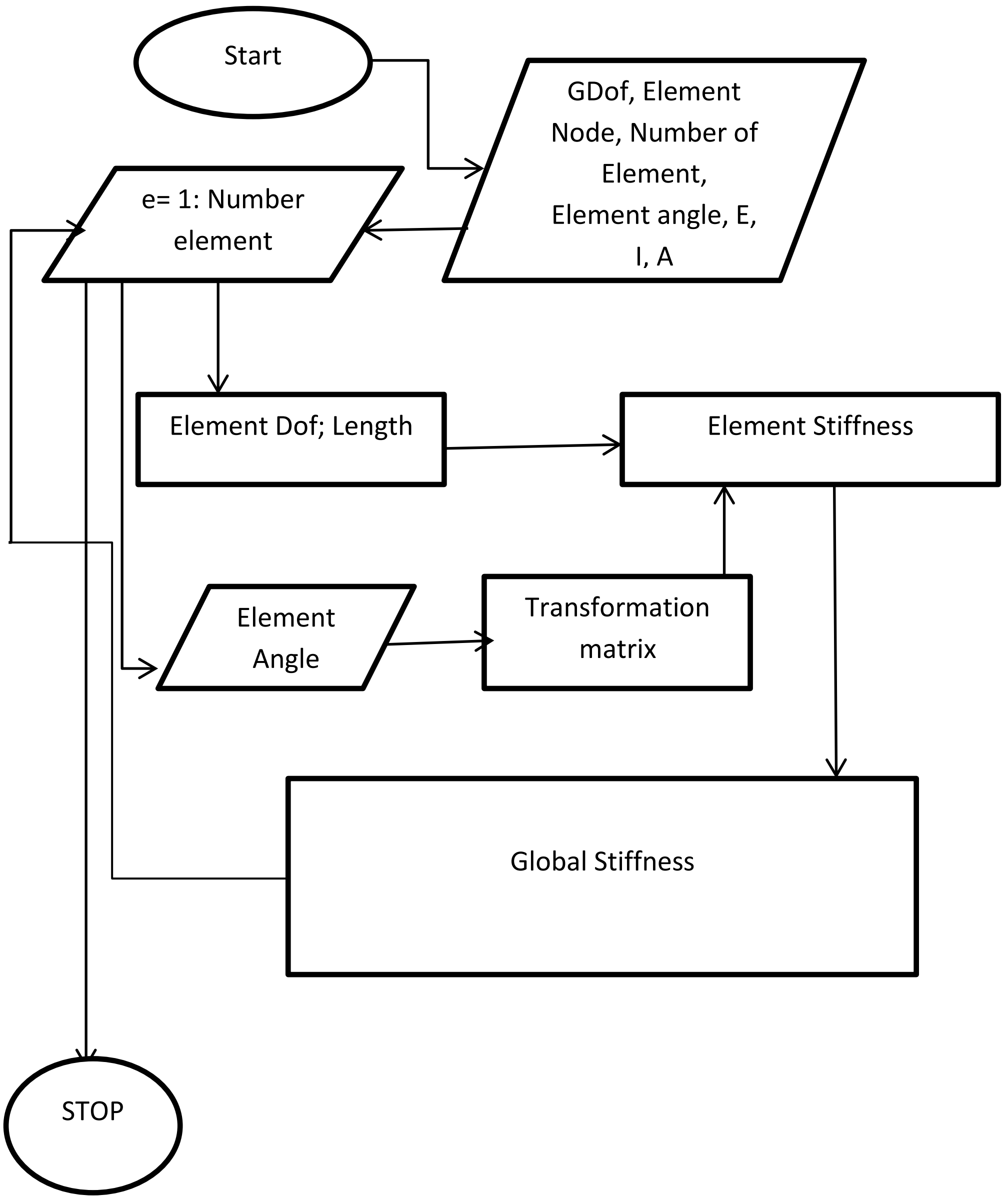
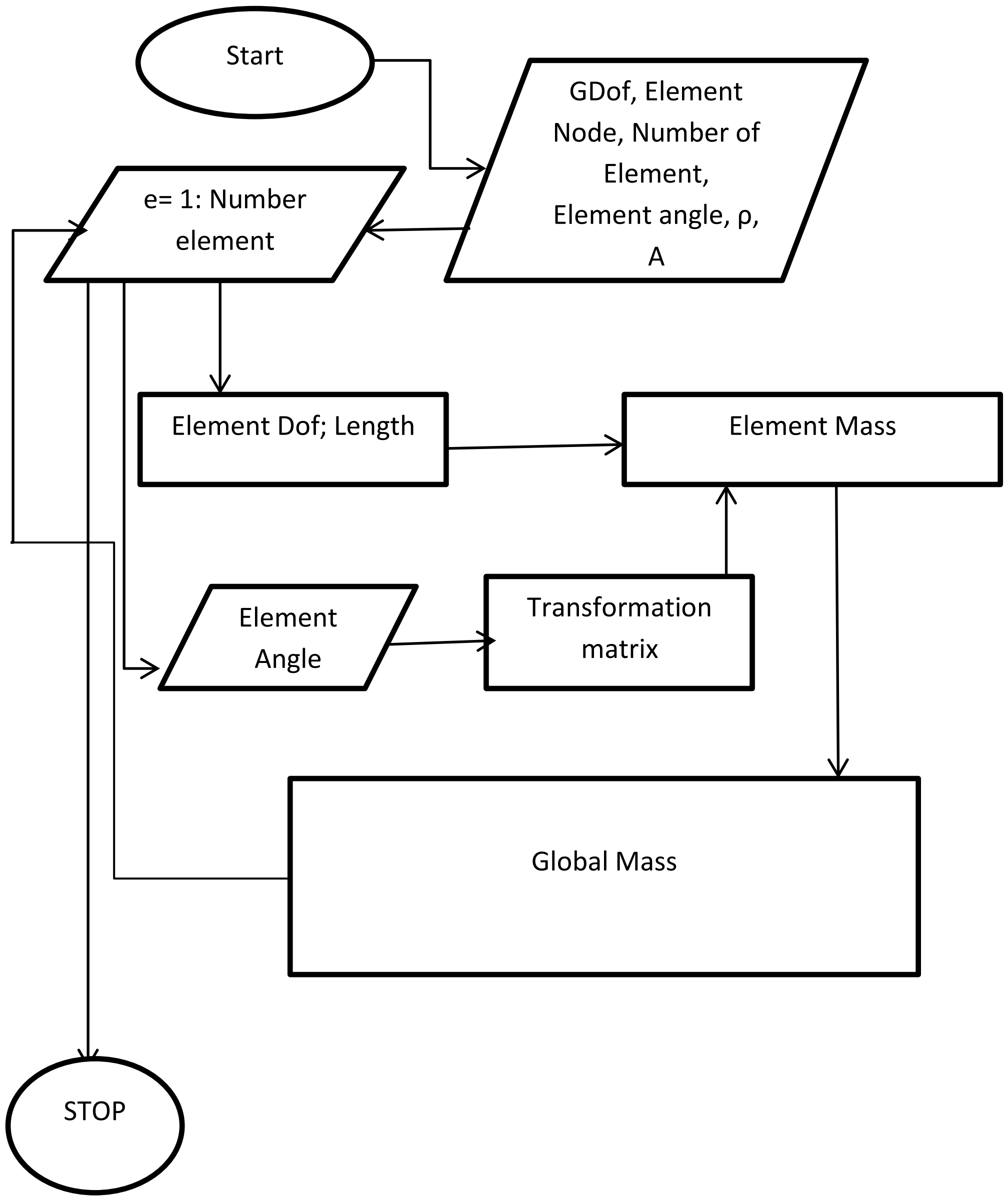


Fig 4.3 Function Mass



Chapter-5

RESULTS

Problem 5.1:- cantilever beam. [Petyt, example 3.7]

Solution:-

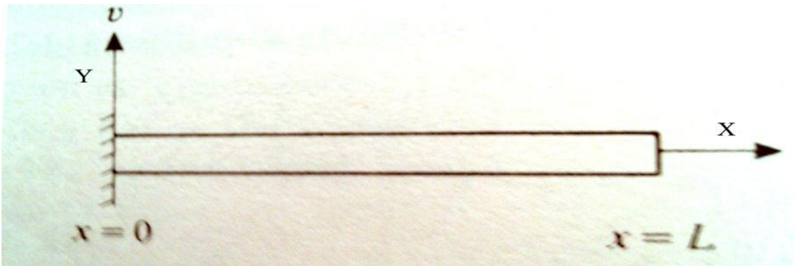


Fig 5.1

Eigen Values (λ) = $\begin{Bmatrix} 0.0594 \\ 5.7691 \end{Bmatrix}$ Frequencies= $(210\lambda)^{.5} * (\frac{EI}{\rho AL4})$ = $\begin{pmatrix} 3.5327 \\ 34.8069 \end{pmatrix} * (\frac{EI}{\rho AL4})$

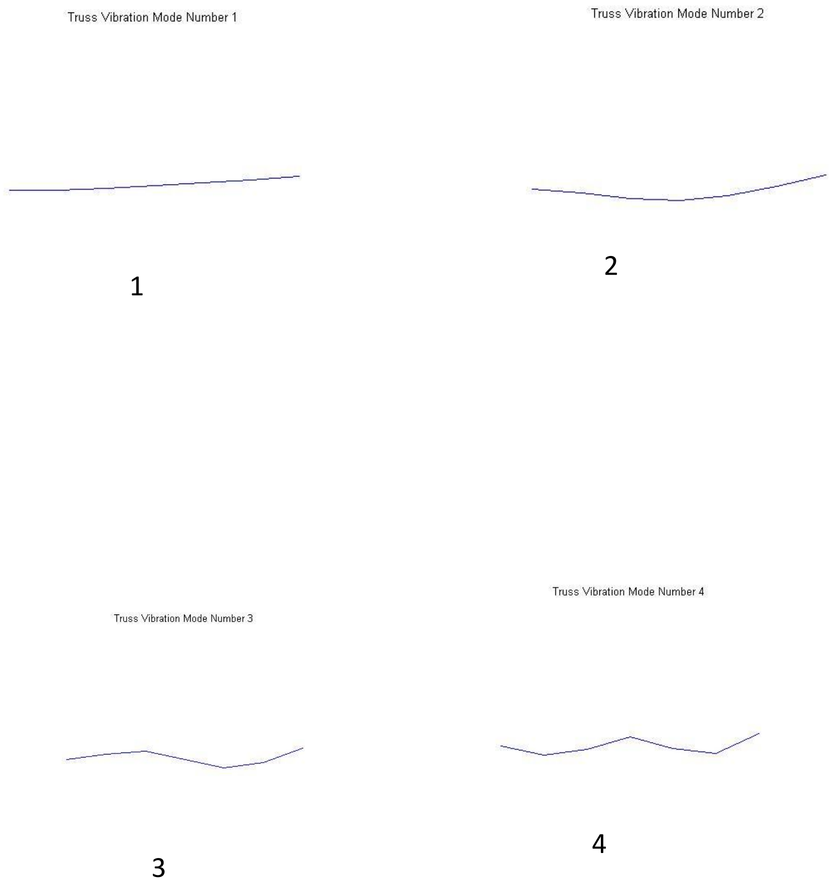
Approximate Solution	Exact Solution	Error%
3.533	3.516	0.484%
34.807	22.035	57.962%

Table 5.1 Comparison between Approximate and exact solution

Mode	1	2	3	4	5	Analytical
1	256.2585	255.1693	255.0718	255.0543	255.0494	255.047649
2		1611.909	1603.595	1600.208	1599.145	1598.740523

Table 5.2 Convergence

Fig 5.2 Mode Shapes



Problem 5.2:- Single bay two-storied frame. E=206.89 GN/m^2; ρ=7.83x10^3 Kg/m^3. [Petyt, Example 3.8]

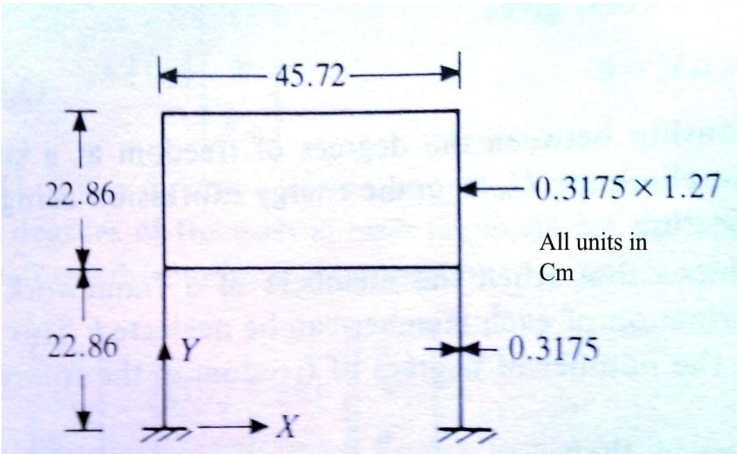


Fig 5.3

Solution:-

Idealization of half framework and discretization

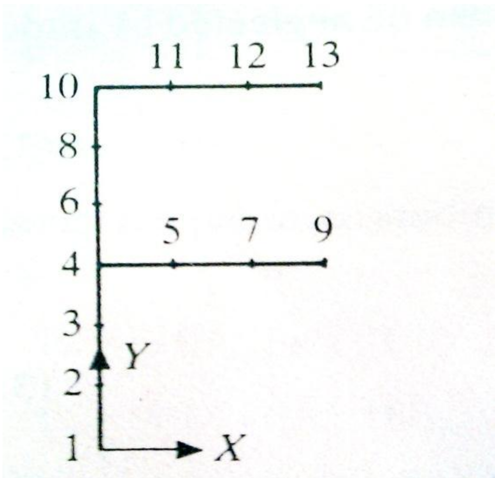
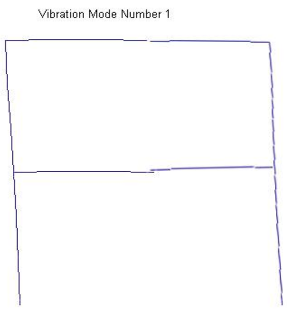


Fig-5.4

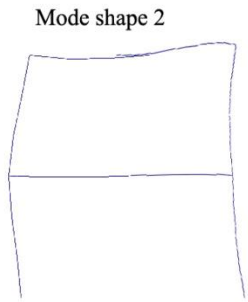
Mode	Frequency(FEM) (Hz)	Frequency (Analytical) (Hz)	Error (%)
1.	**15.14251**	**15.14**	**0.00**
2.	**53.32064**	**53.32**	**0.00**
3.	**155.4666**	**155.31**	**0.10**
4.	**186.5047**	**186.23**	**0.15**
5.	**270.8234**	**270.07**	**0.29**
6.	**347.184**	**-**	**-**
7.	**598.8466**	**-**	**-**
8.	**662.8994**	**-**	**-**
9.	**811.2423**	**-**	**-**
10.	**922.4838**	**-**	**-**
11.	**1454.966**	**-**	**-**
12.	**1571.621**	**-**	**-**
13.	**1855.862**	**-**	**-**
14.	**2069.299**	**-**	**-**
15.	**2730.581**	**-**	**-**
16.	**3000.664**	**-**	**-**
17.	**3578.054**	**-**	**-**
18.	**3999.394**	**-**	**-**
19.	**4927.609**	**-**	**-**
20.	**5409.795**	**-**	**-**
21.	**6118.65**	**-**	**-**
22.	**6404.432**	**-**	**-**
23.	**9810.758**	**-**	**-**
24.	**11044.7**	**-**	**-**
25.	**21498.45**	**-**	**-**
26.	**23870.92**	**-**	**-**
27.	**34480.95**	**-**	**-**
28.	**35408.57**	**-**	**-**

Table 5.3 NATURAL FREQUENCIES

Fig 5.5 MODE SHAPES



1



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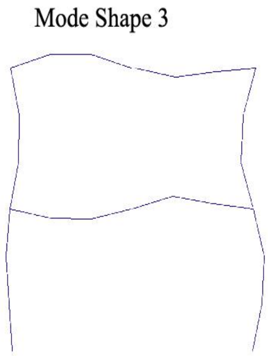

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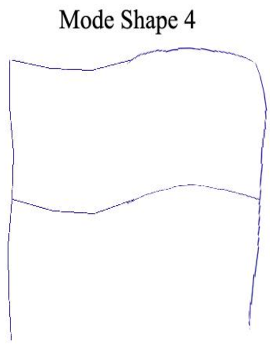

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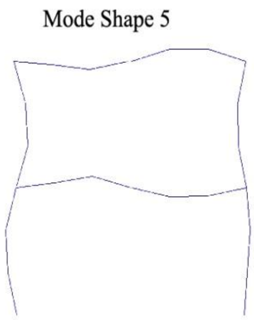


5

Problem 5.3: Frame with inclined member. [Mario Paz, Example 15.1]

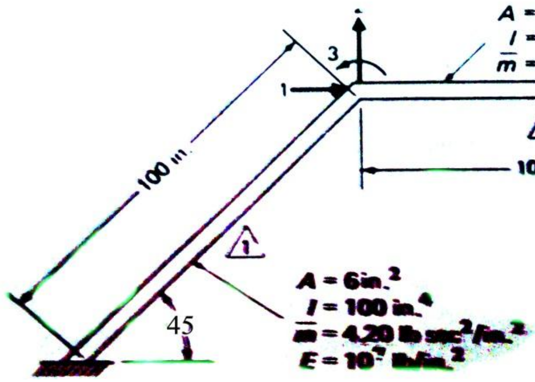


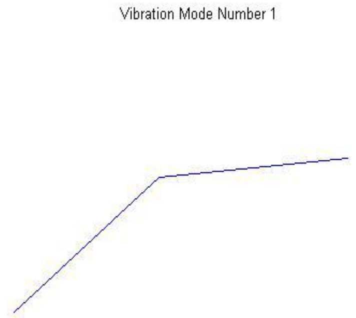
Fig 5.6

Solution:-

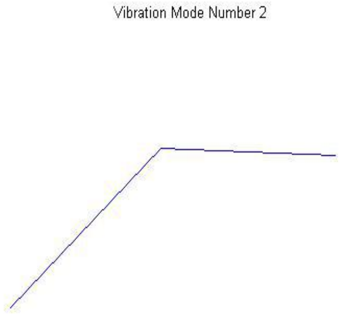
1	25.26865 rad/sec
2	31.25032 rad/sec
3	64.89682 rad/sec

Table 5.4 NATURAL FREQUENCIES

MODE SHAPES



1



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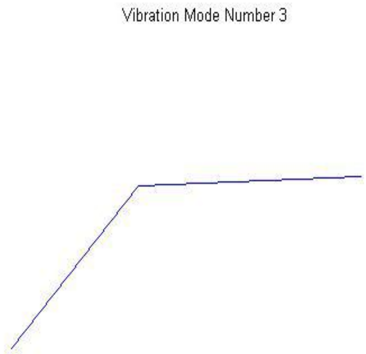


3

Fig 5.7 First three mode shapes

Problem 4.4:- Frame with inclined members. E= 206 GN/m^2; A=$24x10^{-4}$ m^2; I= $48x10^{-8}$ m^4.[Petyt, Problem 3.15]

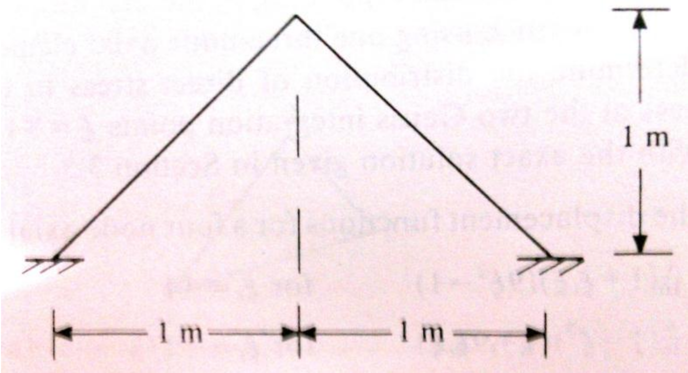
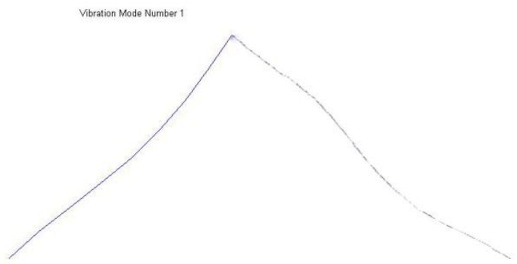


Fig 5.8

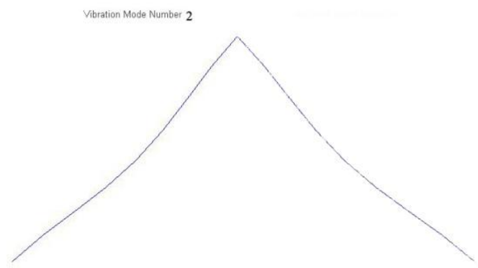
Solution:-

Mode number	FEM solution (Hz)	Analytical solution (Hz)	Error (%)
1	89.00392	88.9	1.01
2	128.5899	128.6	0.01
3	288.5379	286.9	0.56
4	351.0344	350.9	0.03

Table 5.5:- Natural frequencies



1



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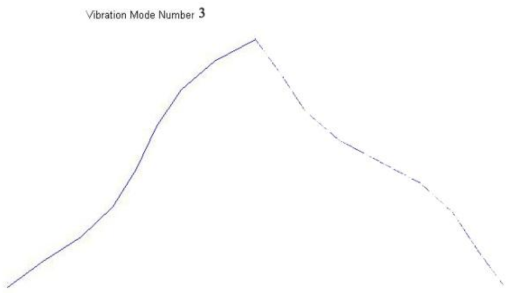

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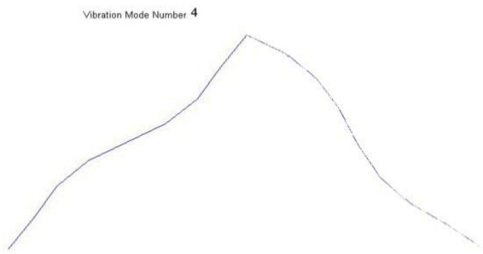

4

Fig 5.9 Mode Shapes

Chapter-6

CONCLUSION AND DISCUSSION

6.1 CONCLUSION

- The FEM results obtained are compared with analytical solutions which compared well.
- The convergence of results was noticed with increased number of elements.
- With this method frames with inclined members as well as straight members can be analyzed.

6.2 DISCUSSION

Finite Element Method is a very useful tool to analyze the frames. Analytical methods require tedious calculations. FEM is a convenient method. One only needs to find matrices and impose boundary conditions. Moreover, FEM can be easily programed into computers. With evolution of computers with faster speed of calculation, FEM has become most widely used in Civil Engineering. No doubt, the modern day software like STAAD Pro, Ansys etc. depend heavily on FEM.

The values of Natural Frequencies obtained from FEM, was of high accuracy. However, the error was high in a problem, because very less number of elements was used. With increment in discretization, results converged to those of analytical solutions.

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